Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over

Pearson

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1. The curve C has equation

$$y = x^2 \cos\left(\frac{1}{2}x\right) \qquad 0 < x \leqslant \pi$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = 2\arctan\left(\frac{4}{x}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = 2\arctan\left(\frac{4}{x_n}\right) \qquad x_1 = 2$$

(b) find the value of x_2 and the value of x_6, giving your answers to 3 decimal places.

(3)

1.a) $y = x^2 \cos\left(\frac{x}{2}\right) \qquad 0 < x \leq \pi$

At stationary points, $\frac{dy}{dx} = 0$

$\boxed{\text{PRODUCT RULE : } y = uv \quad y' = u'v + uv'}$

$u = x^2 \qquad \frac{du}{dx} = 2x$

$v = \cos\left(\frac{x}{2}\right) \qquad \frac{dv}{dx} = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

$\frac{dy}{dx} = 2x\left(\cos\left(\frac{x}{2}\right)\right) + x^2\left(-\frac{1}{2}\sin\left(\frac{x}{2}\right)\right)$

at P → $\frac{dy}{dx} = 0 \quad \therefore \quad 2x\cos\left(\frac{x}{2}\right) - \frac{x^2}{2}\sin\left(\frac{x}{2}\right) = 0$

$2\cancel{x}\cos\left(\frac{x}{2}\right) = \frac{x^{\cancel{2}}}{2}\sin\left(\frac{x}{2}\right)$

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Question 1 continued

$$\tan\left(\frac{x}{2}\right) = \frac{4}{x}$$

$$x = 2\arctan\left(\frac{4}{x}\right)$$

b) $x_{n+1} = 2\arctan\left(\frac{4}{x_n}\right)$

$x_1 = 2$

$x_2 = x_{1+1} = 2\arctan\left(\frac{4}{2}\right) = 2.214$

$x_3 = 2.130$ $\quad$ $x_5 = 2.150$

$x_4 = 2.163$ $\quad$ $x_6 = 2.155$

Q1

(Total 7 marks)

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2. (a) Show that

$$\frac{1-\cos 2x}{2\sin 2x} \equiv k\tan x \qquad x \neq (90n)° \qquad n \in \mathbb{Z}$$

where k is a constant to be found.

(3)

(b) Hence solve, for $0 < \theta < 90°$

$$\frac{9(1-\cos 2\theta)}{2\sin 2\theta} = 2\sec^2\theta$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

2.a) LHS : $\frac{1-\cos(2x)}{2\sin(2x)}$ RHS : $k\tan(x)$

USING COMPOUND ANGLE FORMULAE → $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$= \frac{1-(\cos^2(x)-\sin^2(x))}{2(2\sin(x)\cos(x))}$

$= \frac{1-\cos^2(x)+\sin^2(x)}{4\sin(x)\cos(x)}$ $\cos^2(A)+\sin^2(A) = 1$

$1-\cos^2(A) = \sin^2(A)$

$= \frac{2\sin^{\cancel{2}}(x)}{4\cancel{\sin(x)}\cos(x)} = \frac{1}{2}\tan(x)$

RHS = $k\tan(x)$ = LHS $\therefore k = \frac{1}{2}$

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Question 2 continued

b) $\dfrac{9(1-\cos(2\theta))}{2\sin(2\theta)} = 2\sec^2(\theta)$

$\sin^2 A + \cos^2 A = 1$

divide through by $\cos^2 A$

$9\left(\frac{1}{2}\tan(\theta)\right) = 2\sec^2(\theta)$

$\dfrac{\sin^2 A}{\cos^2 A} + \dfrac{\cos^2 A}{\cos^2 A} = \dfrac{1}{\cos^2 A}$

$\frac{9}{2}\tan(\theta) = 2(\tan^2(\theta)+1)$

$\boxed{\tan^2 A + 1 = \sec^2 A}$

$9\tan(\theta) = 4\tan^2(\theta) + 4$

let $\tan\theta = t$

$4t^2 - 9t + 4 = 0$

$t = \tan(\theta) = \dfrac{9 \pm \sqrt{17}}{8}$

$0 < \theta < 90°$

$\theta = 58.6° \cup 31.4°$

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3. (i) Find

$$\int \frac{12}{(2x-1)^2}\,dx$$

giving your answer in simplest form.

(2)

(ii) (a) Write $\frac{4x+3}{x+2}$ in the form

$A + \frac{B}{x+2}$ where A and B are constants to be found

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$$

giving your answer in simplest form.

(6)

3.(i) $\int \frac{12}{(2x-1)^2}\,dx$

$u = 2x-1 \quad \frac{du}{dx} = 2 \rightarrow dx = \frac{du}{2}$

$\int \frac{12}{u^2} \times \frac{du}{2}$

$= \int 6u^{-2}\,du$

$= -6u^{-1} + c$

$= \frac{-6}{2x+1} + c$

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Question 3 continued

(ii) a) $\frac{4x+3}{x+2}$

$= \frac{4(x+2)-5}{x+2}$

$= 4 - \frac{5}{x+2}$ $A = 4$, $B = -5$

b) $\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$

$= \int_{-8}^{-5} 4 - \frac{5}{x+2}\,dx$

$= \left[4x - 5\ln|x+2|\right]_{-8}^{-5}$

$= 4(-5) - 5\ln|-3| - 4(-8) + 5\ln|-6|$

$= 12 - 5\ln(3) + 5\ln(6)$

$= 12 + 5\ln\left(\frac{6}{3}\right)$ $\boxed{\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)}$

$= 12 + 5\ln(2)$

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4. The functions f and g are defined by

$$f(x) = \frac{4x+6}{x-5} \qquad x \in \mathbb{R},\ x \neq 5$$

$$g(x) = 5 - 2x^2 \qquad x \in \mathbb{R},\ x \leqslant 0$$

(a) Solve the equation

$$fg(x) = 3$$

(4)

(b) Find f^{-1}

(3)

(c) Sketch and label, on the same axes, the curve with equation $y = g(x)$ and the curve with equation $y = g^{-1}(x)$. Show on your sketch the coordinates of the points where each curve meets or cuts the coordinate axes.

(3)

4. a) $f(x) = \frac{4x+6}{x-5}$ $\quad g(x) = 5-2x^2$

$f(g(x))$

$= f(5-2x^2)$

$= \frac{4(5-2x^2)+6}{5-2x^2-5} = \frac{20-8x^2+6}{-2x^2}$

$= \frac{26-8x^2}{-2x^2}$

$= \frac{13-4x^2}{-x^2} = 3$

$13-4x^2 = 3(-x^2)$

$13-4x^2 = -3x^2$ $\quad x \leq 0$

$x^2 = 13$ $\quad x = \cancel{\pm}\, -\sqrt{13}$

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Question 4 continued

b) to find $f^{-1}(x)$: $f(x) = \frac{4x+6}{x-5}$

① write the function using a "y" and set equal to "x" : $x = \frac{4y+6}{y-5}$

② rearrange to make y the subject : $xy - 5x = 4y + 6$

$y = \frac{5x+6}{x-4}$

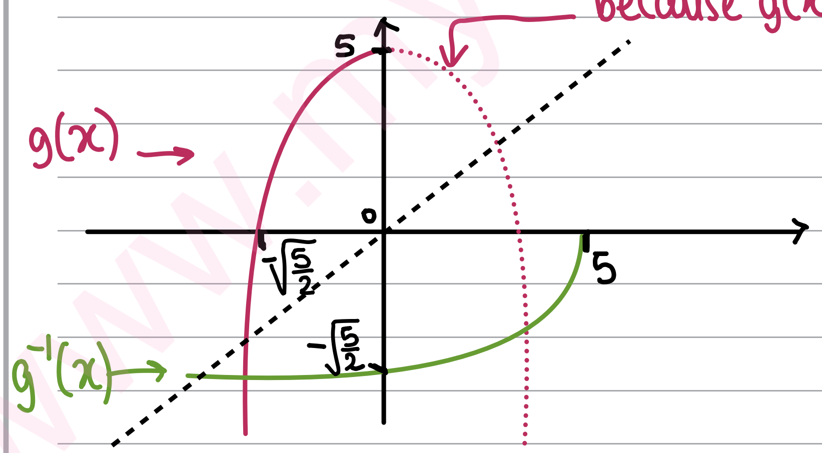
③ replace y with $f^{-1}(x)$:

$f^{-1}(x) = \frac{5x+6}{x-4}$ $x \in R$
$x \neq 4$ ← denominator $\neq 0$

c) $g^{-1}(x)$ is the reflection of $g(x)$ in the line $y = x$

$g(x) = 5 - 2x^2$ ← roots of $g(x) = \pm\sqrt{\frac{5}{2}}$
↳ $x \leq 0$

because $g(x)$ is only for $x \leq 0$



5.

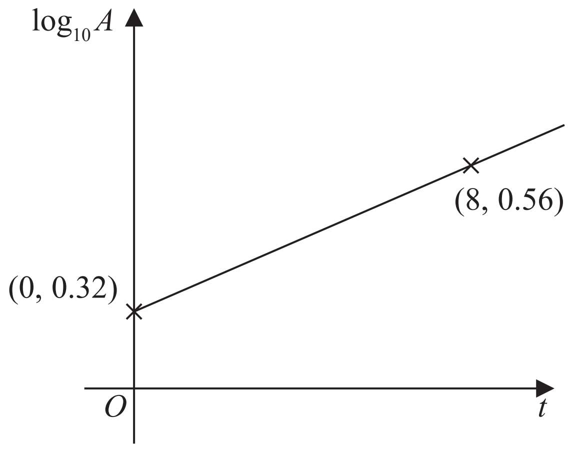


Figure 1

The growth of duckweed on a pond is being studied.

The surface area of the pond covered by duckweed, $A\,\text{m}^2$, at a time t days after the start of the study is modelled by the equation

$$A = pq^t \qquad \text{where } p \text{ and } q \text{ are positive constants}$$

Figure 1 shows the linear relationship between $\log_{10} A$ and t.

The points (0, 0.32) and (8, 0.56) lie on the line as shown.

(a) Find, to 3 decimal places, the value of p and the value of q.

(4)

Using the model with the values of p and q found in part (a),

(b) find the rate of increase of the surface area of the pond covered by duckweed, in m^2/day, exactly 6 days after the start of the study.
Give your answer to 2 decimal places.

(3)

5. a) $A = pq^t$

$\log_{10} A = \log_{10}(pq^t)$

LOG RULES →

$a \log_b(c) = \log_b(c^a)$

$\log_a b + \log_a c = \log_a(bc)$

$\log_a b = c \rightarrow a^c = b$

Leave blank

Question 5 continued

$\log_{10} A = \log_{10}(p) + \log_{10}(q^t)$

$\log_{10} A = \log_{10}(p) + t\log_{10}(q)$

$y = c + x\, m$ ← graph of $\log_{10} A$ against t

known points : $(0, 0.32)$ & $(8, 0.56)$

c-intercept

$\log_{10}(p) = 0.32$

$10^{0.32} = 2.089$

$m = \text{gradient} = \dfrac{0.56 - 0.32}{8 - 0}$

$= 0.03$

$\log_{10}(q) = 0.03$

$q = 10^{0.03}$

$= 1.072$

b) rate of increase of SA $= \dfrac{dA}{dt}$

$A = 2.089 \times 1.072^t$

$y = a^x$ ← rewrite a^x in terms of e

$a = e^{\ln(a)}$

$\frac{d}{dx}(e^{kx}) = ke^{kx}$

$\therefore\ y = (e^{\ln(a)})^x \rightarrow \dfrac{dy}{dx} = \ln(a) \times e^{(\ln a)x} = \ln(a) \times a^x$

$\therefore\ \dfrac{dA}{dt} = 2.089 \times \ln(1.072) \times 1.072^t$

↳ when $t = 6 \rightarrow \dfrac{dA}{dt} = 2.089 \times \ln(1.072) \times 1.072^6$

$= 0.22\ m^2$ per day

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6. Given that k is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left|2x - \dfrac{k}{3}\right|$

Show on each sketch the coordinates, in terms of k, of each point where the graph meets or cuts the axes.

(4)

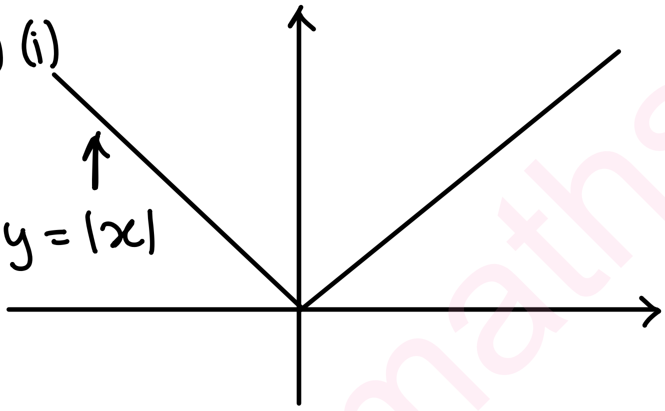
(b) Hence find, in terms of k, the values of x for which

$$\left|2x - \frac{k}{3}\right| = k - 2|x|$$

giving your answers in simplest form.

(4)

6. a) (i)



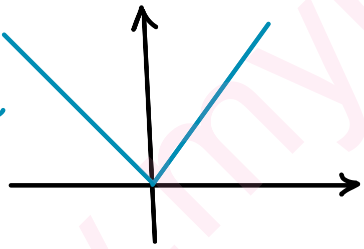
reflection in x-axis

$y = k - 2|x|$

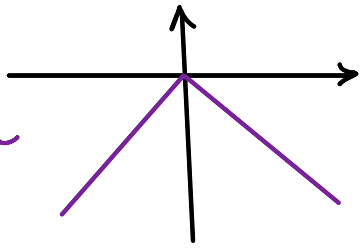
translation through the vector $\binom{0}{k}$

stretch scale factor 2 parallel to y axis

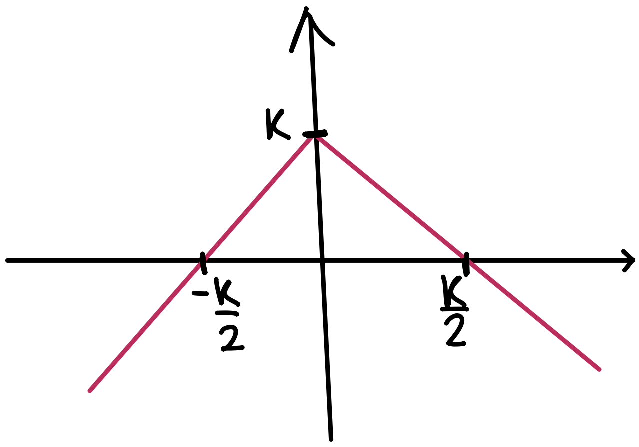
① stretch



② reflection



③ translation



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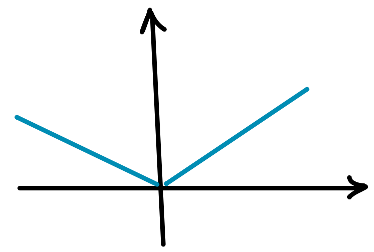
Question 6 continued

(ii) $y = \left| 2x - \frac{K}{3} \right|$

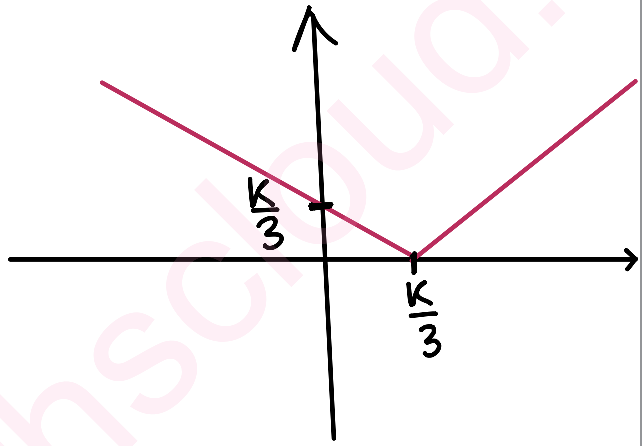
stretch scale factor $\frac{1}{2}$ parallel to x-axis

translation through the vector $\binom{K/3}{0}$

① stretch



② translation



b) $\left| 2x - \frac{K}{3} \right| = K - 2|x|$

$-2x + \frac{K}{3} = K + 2x$ OR $2x - \frac{K}{3} = K - 2x$

$4x = -\frac{2K}{3}$ $\qquad$ $4x = \frac{4K}{3}$

$x = -\frac{K}{6}$ $\qquad$ $x = \frac{K}{3}$

Leave blank

7. Given that

$$x = 6\sin^2 2y \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{(Bx - x^2)}}$$

where A and B are integers to be found.

(5)

7. a) $x = 6\sin^2(2y) \qquad 0 < y < \frac{\pi}{4}$

$\boxed{\text{CHAIN RULE : } \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$

$v = \sin(2y) \qquad \frac{dv}{dy} = 2\cos(2y)$

$x = 6(\sin(2y))^2$

$= 6v^2$

$\frac{dx}{dv} = 12v \qquad \frac{dx}{dy} = \frac{dx}{dv} \times \frac{dv}{dy}$

$= 12v \times 2\cos(2y)$

$= 24\sin(2y)\cos(2y)$

$x = 6\sin^2(2y)$

$\sin(2y) = \sqrt{\frac{x}{6}} \qquad \cos(2y) = \sqrt{1 - \sin^2(2y)} = \sqrt{1 - \frac{x}{6}}$

$= \sqrt{\frac{6-x}{6}}$

$\therefore \frac{dx}{dy} = 24\sqrt{\frac{x}{6}}\sqrt{\frac{6-x}{6}} = \frac{24}{6}\sqrt{x(6-x)}$

$= 4\sqrt{x(6-x)}$

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Question 7 continued

$\boxed{\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}}$

$\therefore \frac{dy}{dx} = \frac{1}{4\sqrt{6x - x^2}}$

$A = 4$
$B = 6$

Q7

(Total 5 marks)

8. A scientist is studying a population of fish in a lake. The number of fish, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{600e^{0.3t}}{2 + e^{0.3t}} \qquad t \geqslant 0$$

Use the equation of the model to answer parts (a), (b), (c), (d) and (e).

(a) Find the number of fish in the lake at the start of the study.

(1)

(b) Find the upper limit to the number of fish in the lake.

(1)

(c) Find the time, after the start of the study, when there are predicted to be 500 fish in the lake. Give your answer in years and months to the nearest month.

(4)

(d) Show that

$$\frac{dN}{dt} = \frac{Ae^{0.3t}}{(2 + e^{0.3t})^2}$$

where A is a constant to be found.

(3)

Given that when $t = T$, $\frac{dN}{dt} = 8$

(e) find the value of T to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

8.a) $N = \dfrac{600e^{0.3t}}{2+e^{0.3t}}$

when $t=0$ $\quad N_0 = \dfrac{600e^{0.3(0)}}{2+e^{0.3(0)}} = \dfrac{600}{2+1} = 200$

b) $N = \dfrac{600e^{0.3t}}{2+e^{0.3t}}$ $\leftarrow$ divide through by $e^{0.3t}$

$N = \dfrac{600}{\frac{2}{e^{0.3t}} + 1}$ $\quad$ as $t \to \infty$ $\quad e^{0.3t} \to \infty \quad \therefore \frac{2}{e^{0.3t}} \to 0$

Question 8 continued

as $t \to \infty$ $\quad N = \dfrac{600}{\cancel{\dfrac{2}{e^{0.3t}}}^{\,0} + 1}$ $\quad \therefore\ N \to 600$

c) $500 = \dfrac{600e^{0.3t}}{2+e^{0.3t}}$

$500(2+e^{0.3t}) = 600e^{0.3t}$

$1000 + 500e^{0.3t} = 600e^{0.3t}$

$e^{0.3t} = 10$

$t = \dfrac{\ln(10)}{0.3} = 7.68$ yrs

$= 7$ yrs 8 months

d) Quotient rule for differentiating : $y = \dfrac{u}{v} \to \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$N = \dfrac{600e^{0.3t}}{2+e^{0.3t}}$ $\quad u = 600e^{0.3t}$ $\quad \dfrac{du}{dt} = 180e^{0.3t}$

$v = 2+e^{0.3t}$ $\quad \dfrac{dv}{dt} = 0.3e^{0.3t}$

$\dfrac{dN}{dt} = \dfrac{2+e^{0.3t}(180e^{0.3t}) - 600e^{0.3t}(0.3e^{0.3t})}{(2+e^{0.3t})^2}$

$= \dfrac{360e^{0.3t}}{(2+e^{0.3t})^2}$ $\quad A = 360$

e) $t = T$

$\dfrac{dN}{dt} = \dfrac{360e^{0.3T}}{(2+e^{0.3T})^2} = 8$

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Question 8 continued

$360e^{0.3T} = 8(2+e^{0.3T})^2$

$360e^{0.3T} = 8(4 + 4e^{0.3T} + e^{0.6T})$

$360e^{0.3T} = 32 + 32e^{0.3T} + 8e^{0.6T}$

$e^{0.6T} - 41e^{0.3T} + 4 = 0$

let $e^{0.3T} = a$

$a^2 - 41a + 4 = 0$

$a = e^{0.3T} = \dfrac{41 \cancel{\pm} 3\sqrt{185}}{2}$

reject solution ≤ 1 as T is positive

$T = \frac{10}{3}\ln\left(\frac{41+3\sqrt{185}}{2}\right)$

$= 12.4$ yrs

Leave blank

9. (a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians, to 3 decimal places.

(3)

The function g is defined by

$$g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \qquad \theta > 0$$

Find

(b) (i) the minimum value of $g(\theta)$

(ii) the smallest value of θ at which the minimum value occurs.

(3)

The function h is defined by

$$h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$$

(c) Find the range of h.

(2)

9.a) $12\sin(x) - 5\cos(x)$

$= R\sin(x-\alpha)$ ← using compound angle formulae $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$R(\sin x\cos\alpha - \cos x\sin\alpha)$

Compare expanded expression to given

$R\sin x\cos\alpha - R\cos x\sin\alpha = 12\sin(x) - 5\cos(x)$

$R\cos\alpha = 12 \qquad R\sin\alpha = 5$

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \tan\alpha = \frac{5}{12}$

$\alpha = 0.395$

$(R\cos\alpha)^2 + (R\sin\alpha)^2$ $\cos^2 A + \sin^2 A = 1$ identity

$= R^2(\cos^2\alpha + \sin^2\alpha) = R^2(1)$

$= 12^2 + 5^2$

$\therefore R^2 = 169 \qquad R = \cancel{\pm}13$

↑ reject negative $R > 0$

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Question 9 continued

b)(i) $g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \quad \theta > 0$

$= 10 + 13\sin\left(2\theta - \frac{\pi}{6} - 0.395\right)$

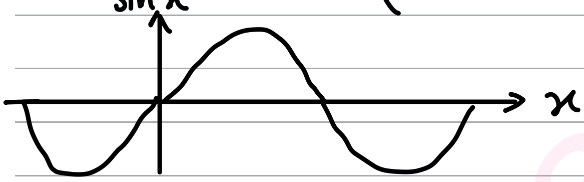
min value of $\quad$ = 13 × min value of $\sin(\ldots)$

$\sin_{min} = -1$

$\therefore g(\theta)_{min} = 10 - 13 = -3$

(ii) value of θ at which occurs:

when $\sin\left(2\theta - \frac{\pi}{6} - 0.395\right) = -1$



$2\theta - \frac{\pi}{6} - 0.395 = -\frac{\pi}{2} \cup \frac{3\pi}{2} \cup \ldots$

try with $-\frac{\pi}{2} \rightarrow \cancel{\theta = -0.326}$ $\quad \theta > 0$

$\therefore$ try with $\frac{3\pi}{2} \rightarrow \theta = 2.82$

c) $h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$

$= 10 - (13\sin\beta)^2$

range of $13\sin\beta$: $-13 \leq 13\sin\beta \leq 13$

range of $(13\sin\beta)^2$: $0 \leq (13\sin\beta)^2 \leq 169$

$\therefore$ range of $h(\beta)$: $\boxed{-159 \leq h(\beta) \leq 10}$